TECHNICAL NOTE

Correlations for free convection and surface radiation in a square cavity

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In the present study the results of a numerical investigation of combined surface radiation and free convection in a square cavity with air as the intervening medium are reported. The computations have been performed for $10^3 \leq Gr \leq 10^6$, with the emissivities of all the walls varying between 0 and 1. Surface radiation reduces the convective heat transfer across the cavity and at the same time contributes to the overall heat transfer by direct radiant heat transfer across the cavity. This "dual" nature of radiation has been qualitatively highlighted in an earlier investigation by the authors. The objective of the present study however, is to give comprehensive correlations for convection and radiation based on the numerical calculations of the coupled problem. The basis for the choice of the form of the various terms in the correlations has also been brought out.

Keywords: radiation convection interaction; radiation Nusselt number; convection Nusselt number; correlation

Mathematical formulation and solution procedure

The governing equations for two-dimensional (2-D), laminar, constant property fluid with Boussinesq approximation for free convection in a square cavity are the well-known Navier–Stokes equations. In the present study, the vorticity-stream-function form of the above equations is used. The governing equations, along with the boundary conditions for a cavity with sidewall heating and adiabatic top and bottom walls, are given in a number of references (see, for example, Balaji and Venkateshan 1993). The advantage of the vorticity formulation is the reduction of two momentum equations into one vorticity transport equation along with the elimination of the pressure terms. The boundary conditions for the stream function and vorticity on the four solid walls are well established (Gosman et al. 1969).

However, for the sake of completeness, the boundary conditions on temperature will be elaborated, since the coupling between radiation and free convection arises only in the temperature on the walls. The details of the problem geometry are given in Figure 1. The left wall is an isothermal heat source at temperature T_H, and the right wall is an isothermal heat sink at T_C. The top and bottom walls "float" at a temperature governed by a balance between convection and radiation. Stated more explicitly, the sum of the convective and radiative fluxes on each of these two walls is zero. Mathematically the coupling between free convection and radiation on these two walls can be represented as

$$-k\,\partial T/\partial x + (J - I) = 0 \tag{1}$$

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J and I represent the elemental radiosities and irradiations. If subscript i denotes the element under consideration, then

$$J_i = \varepsilon_i \sigma T_i^4 + (1 - \varepsilon_i) I_i \tag{2}$$

$$I_i = \sum_{j=1}^{N} F_{ij} J_j \tag{3}$$

where N is the number of surface elements (depending on the grid size).

In nondimensional form, Equation 1 becomes

$$\partial\phi/\partial X = N_{RC}(j - i) \tag{4}$$

With reference to the solution procedure for convection, a standard finite-volume method based on Gosman et al. (1969) was used. A 21 × 21 nonuniform grid with grid clustering near the walls was used for the convection solver. To ensure grid compatibility, the same grids were retained for radiation calculations. The standard enclosure method was used for the radiation equations. In the present case, there are 20 zones on each wall, and the whole enclosure has 80 zones. The view

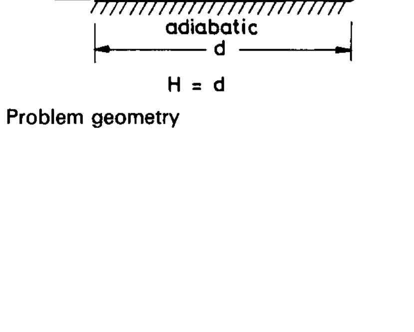

Figure 1 Problem geometry

factors are evaluated by Hottel's cross-string method (Hottel and Saroffim 1967). The required radiosities are obtained by the Gauss–Seidel method.

For every iteration of the convection solver, the radiosities and irradiations were evaluated, and Equation 4 was used to obtain new temperatures on the top and bottom walls. The above procedure was repeated until temperatures converged to 0.05%.

Results and discussion

Calculations were made for a wide range of parameters, shown in Table 1. Grid-sensitivity analysis was done for both convection and radiation. For a typical case when the ε of all the walls was 1, $\mathrm{Gr} = 5 \times 10^5$ and $T_R = 0.73$ the difference in $\overline{\mathrm{Nu}}_C$ between a 21 × 21 grid and a 31 × 31 grid was 2%, and that for $\overline{\mathrm{Nu}}_R$ was 0.05%. Hence, in the present study, calculations were done with a 21 × 21 grid.

Table 1 Range of parameters in the present study

$10^3 \le \mathrm{Gr} \le 10^6$
$0 \le \varepsilon_H \le 1$
$0 \le \varepsilon_C \le 1$
$0 \le \varepsilon_B \le 1$
$0 \le \varepsilon_T \le 1$
$0.73 \le T_R \le 0.95$
$4 \le N_{RC} \le 22$

Note: Actual values of ε used in the calculations for all the walls: (0, 0.3, 0.5, 0.7, 1).

Convection

Based on a large number of numerical data (55 points), $\overline{\mathrm{Nu}}_C$ correlates as

$$\overline{\mathrm{Nu}}_C = 0.149\,\mathrm{Gr}^{0.294}(1+\varepsilon_H)^{-0.279}(1+\varepsilon_C)^{0.182}(1+\varepsilon_B)^{-0.135}(1+\varepsilon_T)^{0.115}(N_{RC}/(N_{RC}+1))^{0.272} \quad (5)$$

The above correlation has a correlation coefficient of 0.998; the maximum error between data and correlation was 4.9%. The correlation was chosen in its present form after a careful consideration of the effect of each of the parameters on $\overline{\mathrm{Nu}}_C$. When surface radiation is considered, $\overline{\mathrm{Nu}}_C$ becomes a function of radiative parameters apart from Gr. Mathematically,

$$\overline{\mathrm{Nu}}_C = a\mathrm{Gr}^b f(\varepsilon_H, \varepsilon_C, \varepsilon_B, \varepsilon_T, T_R, N_{RC}) \quad (6)$$

In the present study, a power law form is used to quantify the effect of the above parameters on $\overline{\mathrm{Nu}}_C$. Even when the emissivity of all the walls is zero, $\overline{\mathrm{Nu}}_C$ will be nonzero and hence the $(1 + \varepsilon)$ form is used in the correlation for the emissivities of the four walls. With reference to N_{RC}, a closer look is indeed revealing. N_{RC} by definition is $\sigma T_H^4 d/k(T_H - T_C)$. In the present study, the fluid considered was air (Pr = 0.71), and so the thermal conductivity (k) is fixed. Hence the only way N_{RC} can change is either by changing the spacing (d) or the temperature level. But when this happens, Gr also changes. Therefore, the important point emerges that N_{RC} actually usurps the role of Gr when a correlation is attempted for convection. Also, the effect of surface radiation is to reduce the convective heat transfer only to the extent of 12%–15% in a square cavity (Balaji and Venkateshan 1993). Hence if a power law form is used for N_{RC}, it will underplay the importance of Gr in determining $\overline{\mathrm{Nu}}_C$. Also, from Table 1, it is clear that N_{RC} is always greater than unity. In consideration of these reasons, the form $(N_{RC}/(N_{RC}+1))$ was chosen. Actually, the correlation coefficient substantially improved when this form was used. Simultaneously, the standard deviation was also substantially reduced. The Gr exponent appearing in the correlation (Equation 5) is 0.294, which is close to the usually quoted value for the cavity problem (Ostrach 1972). With reference to T_R, it can be seen that N_{RC} includes T_R indirectly, since it contains the term $(T_H - T_C)$. Hence T_R is not independently used in the

Notation

a	Constant in the correlation for Nusselt number
b	Exponent of Grashof number in the correlation
d	Spacing, m
f	Function relating radiative parameters with free convection
F_{ij}	View factor from the ith element to the jth element
g	Acceleration due to gravity, m/s^2
Gr	Grashof number based on d, $g\,\beta\,(T_H - T_C)\,d^3/\nu^2$
H	Height of the enclosure, m
i	Nondimensional irradiation, $I/\sigma T_H^4$
I	Elemental irradiation, W/m^2
j	Nondimensional radiosity, $J/\sigma T_H^4$
J	Elemental radiosity, W/m^2
k	Thermal conductivity of fluid, W/m K
N	Number of surface elements used in radiation calculations
N_{RC}	Radiation conduction interaction parameter, $\sigma T_H^4 d/k(T_H - T_C)$
Nu_C	Local convection Nusselt number based on d, $-(\partial\phi)/\partial Y)_{Y=0}$
$\overline{\mathrm{Nu}}_c$	Average or mean convection Nusselt number, $\int_0^2 \mathrm{Nu}_C/2\,dX$
Nu_R	Local radiation Nusselt number based on d, $q_R\,d/k(T_H - T_C)$
$\overline{\mathrm{Nu}}_R$	Average or mean radiation Nusselt number, $\int_0^2 \mathrm{Nu}_R/2\,dX$
Pr	Prandtl number, ν/α
q_R	Elemental radiative heat flux, $(J - I)$, W/m^2
T	Temperature at any location (X, Y), K
T_C	Temperature of right wall, K
T_H	Temperature of left wall, K
T_R	Temperature ratio, T_C/T_H
x	Vertical distance, m
X	Nondimensional vertical distance, $2x/d$
y	Horizontal distance, m
Y	Nondimensional horizontal distance, $2y/d$

Greek symbols

α	Thermal diffusivity, m^2/s
β	Thermal expansion coefficient, 1/K
ε	Emissivity
ν	Kinematic viscosity, m^2/s
ϕ	Nondimensional temperature, $(T - T_C)/(T_H - T_C)$

Subscripts

H	Hot wall
C	Cold wall
T	Top wall
B	Bottom wall

correlation. All the above considerations were motivating factors in choosing the correlation in the present form. If one takes a closer look at the correlation, all the $(1 + \varepsilon)$ terms as well as the $(N_{RC}/(N_{RC} + 1))$ are of the order of unity, whereas the Grashof number is at least three orders of magnitude more than the aforesaid terms. This fact, combined with an exponent of 0.294 for Gr, brings out the strong dependence of $\overline{Nu}_C$ on Gr and the relatively weak dependence of $\overline{Nu}_C$ on radiative parameters. Also, it can be seen that of all the walls, the left wall has the strongest effect on convection. This is intuitively apparent, since the left wall is the only heat source for both convection and radiation. The excellent agreement between the data and correlation can be seen in Figure 2.

Radiation

The radiation Nusselt number $(\overline{Nu}_R)$ can be correlated as

$$\overline{Nu}_R = 0.657\,Gr^{-0.0093}\,\varepsilon_H^{0.808}\varepsilon_C^{0.342}\,(1 + \varepsilon_B)^{0.199}\,(1 + \varepsilon_T)^{-0.039}\,(1 - T_R^4)^{1.149}\,N_{RC}^{1.051} \quad (7)$$

The above correlation has a correlation coefficient of 0.998, and the maximum error between data and correlation was 6.0%. The parameters governing the problem are now clear from the convection results. However, certain finer points are worth mentioning. With reference to $\overline{Nu}_R$, it is quite clear that when $\varepsilon_H = 0$ and when $\varepsilon_C = 0$, then $\overline{Nu}_R$ is 0. Hence a power law form is used for ε_H and ε_C. However, when the emissivity of the top and bottom walls is zero, $\overline{Nu}_R$ can be nonzero. From a physical standpoint, this is basically because the left wall is the only source of radiation in the present problem and the right wall is the only heat sink, since the other two walls are truly adiabatic. Hence the $(1 + \varepsilon)$ form is used for the top and bottom walls. T_R is a crucial parameter for radiation, unlike convection, basically because radiative flux is proportional to $(T_H^4 - T_C^4)$. This expression is actually $T_H^4(1 - T_R^4)$. Hence, the $(1 - T_R^4)$ form is used in correlating $\overline{Nu}_R$. Since T_H^4 appears in N_{RC}, it is also a crucial parameter in determining the radiant Nusselt number. And finally, the link between free convection and radiation appears in the power law form for Gr. The excellent agreement between the data and correlation is seen in Figure 3.

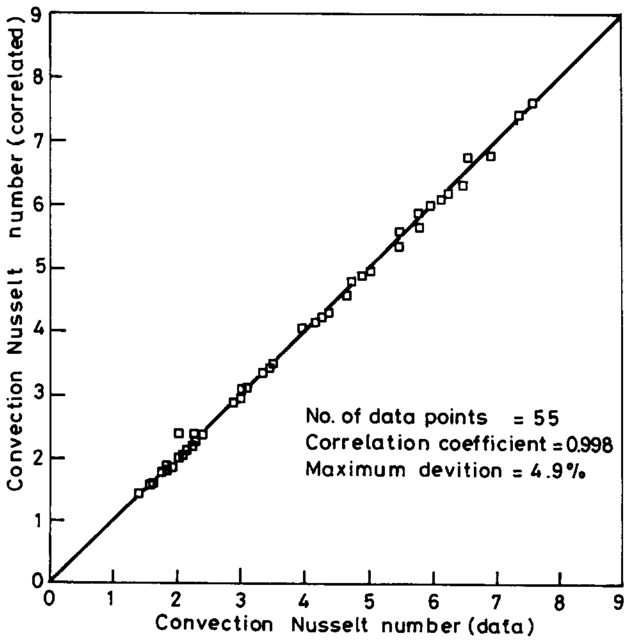

Figure 2 Comparison of $\overline{Nu}_C$ (data) with $\overline{Nu}_C$ (correlated)

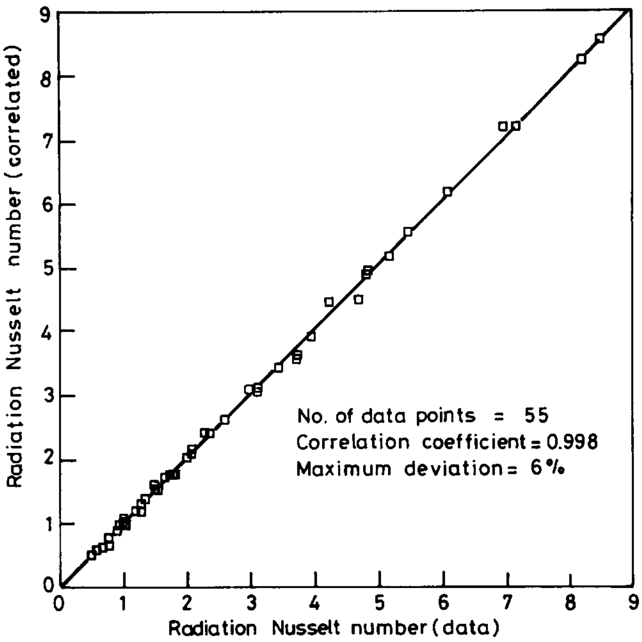

Figure 3 Comparison of $\overline{Nu}_R$ (data) with $\overline{Nu}_R$ (correlated)

The arguments presented above clearly highlight the importance of choosing the form of the correlation that must be consistent with the physical understanding of the phenomenon under question. In fact, the correlation for convection includes the radiative parameters and vice versa, and this is a direct consequence of the coupling in the problem under consideration.

Finally, it is of interest to note that since both $\overline{Nu}_C$ and $\overline{Nu}_R$ are based on the spacing d, the overall Nusselt number, which is the sum of the two, will actually give the effective thermal conductivity of the system. Typically, if $\overline{Nu}_C = 4$ and $\overline{Nu}_R = 2$, then the overall Nusselt number is 6, and if $k = 0.027$ W/m K, then the effective k for the system is $6 \times 0.027 = 0.162$ W/m K. From a physical standpoint, this means that the effect of convection and radiation is equivalent to pure conductive heat transfer across the air layer with a thermal conductivity equal to the effective thermal conductivity.

Conclusions

In the present study, the coupled problem of surface radiation with free convection in a square cavity with air as the medium was numerically solved. Correlations have been developed for both convective as well as radiative heat transfer. The study has brought out the importance of the form of the correlation, which should be consistent with the physics associated with the problem.

References

Balaji, C. and Venkateshan, S. P. 1993. Interaction of surface radiation with free convection in a square cavity. *Int. J. Heat Fluid Flow*, **14** (3), 260–267

Gosman, A. D., Pun, W. M., Runchal, A. K., Spalding, D. B., and Wolfshtein, M. 1969. *Heat and Mass Transfer in Recirculating Flows.* Academic Press, London

Hottel, H. C. and Saroffim, A. F. 1967. *Radiative Heat Transfer.* McGraw-Hill, New York

Ostrach, S. 1972. Natural convection in enclosures. *Advances in Heat Transfer*, Vol. 8. Academic Press, London